Center for Advanced Mesoscience & Nanotechnology

Russian-Chinese International School "Superconducting functional materials for advanced quantum technologies"

Intertype superconductivity in ferromagnetic superconductors

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together with my coauthors

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Outline

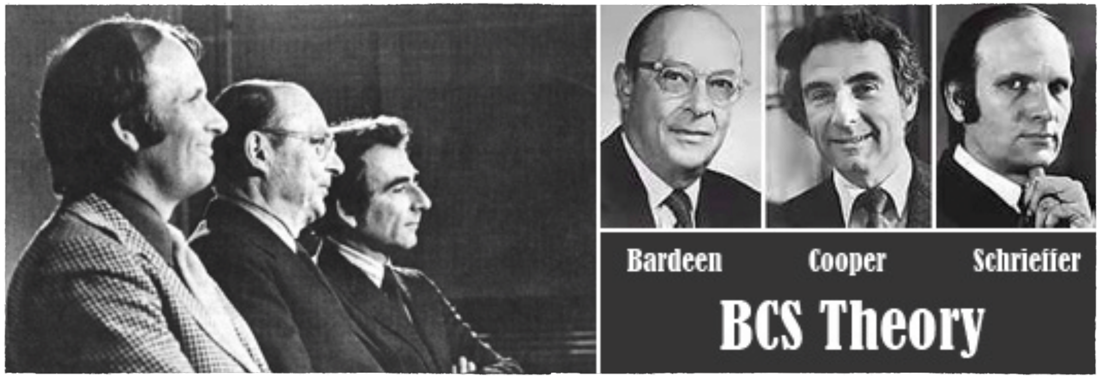
- Ferromagnetic-superconducting pnictides
- Spontaneous pattern formation in pnictides - interplay of magnetism and superconductivity
- Intertype physics and intertype superconductors (short review)
- Minimal theoretical model: the Ginzburg-Landau (GL) theory for the magnetic subsystem plus the extended GL theory for the superconductive subsystem
- Ferromagnetic-superconducting pnictides as intertype superconductors

Ferromagnetic-superconducting pnictides

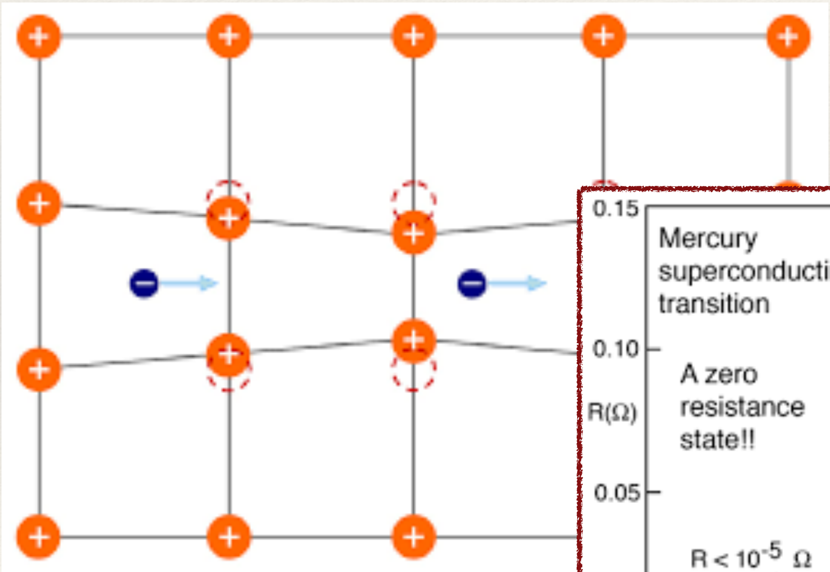
There are materials where we can observe two or more ordered phases, i.e two or more order parameters, which coexist in one system. For example, superconductivity and ferromagnetism.

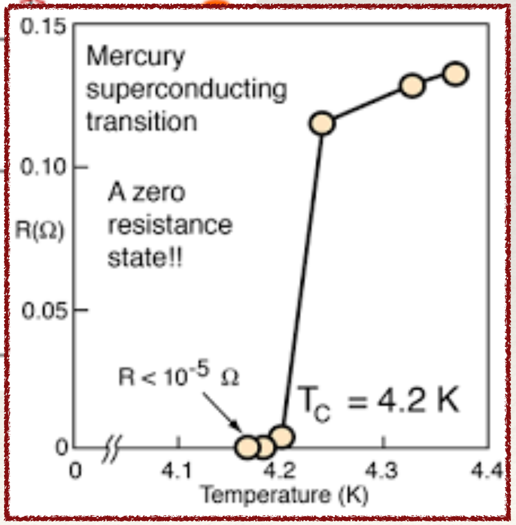
Ferromagnetism is related to the state at which magnetic moments are aligned parallel to each other, and they can remain parallel without external magnetic field; this state appears in materials below the Curie point T_m

Superconductivity is connected with the quantum condensation of the pairs (mostly) of charge carriers, the conventional single-band materials obey the standard BCS theory of superconductivity; the superconducting order appears below the critical temperature T_c

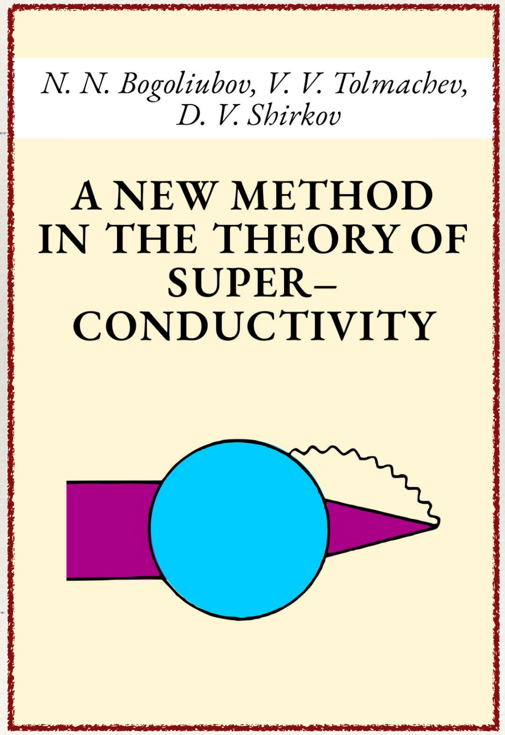


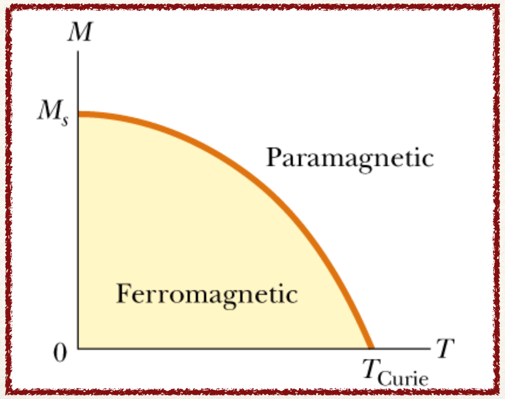
The Baardeen-Cooper-Schrieffer model, at present we work with the Baarden-Cooper-Schrieffer-Bogoliubov Hamiltonian



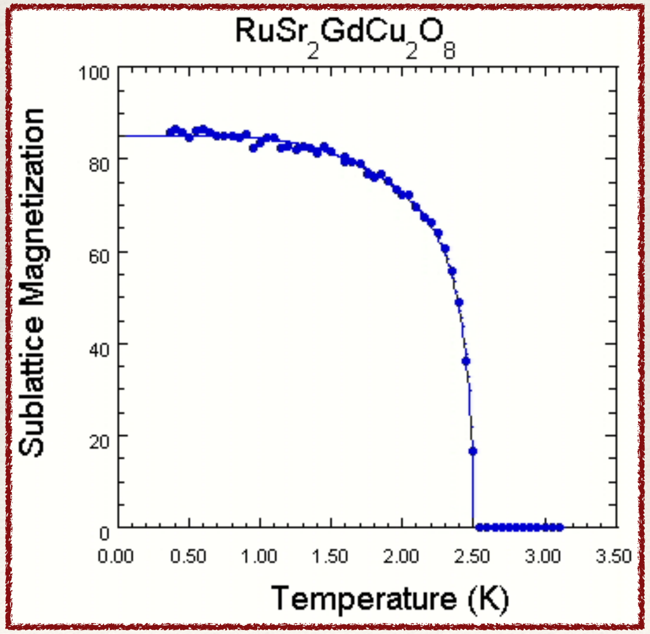


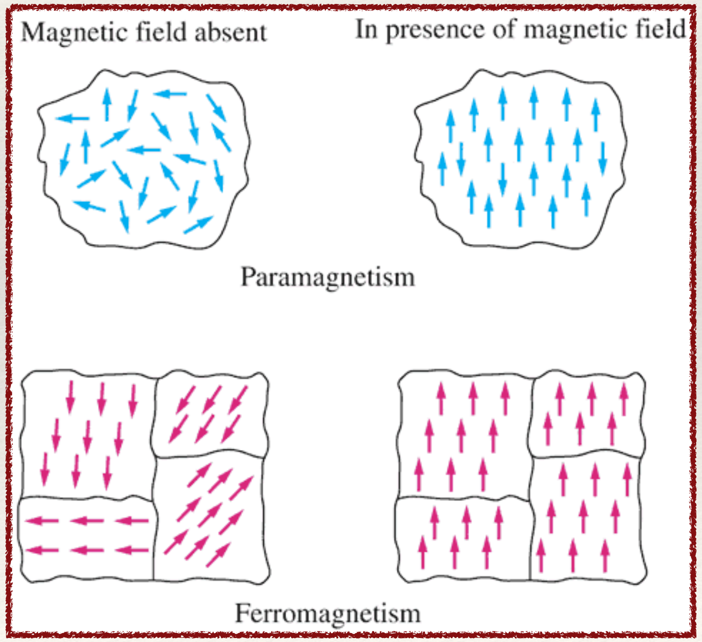
N. N. Bogoliubov

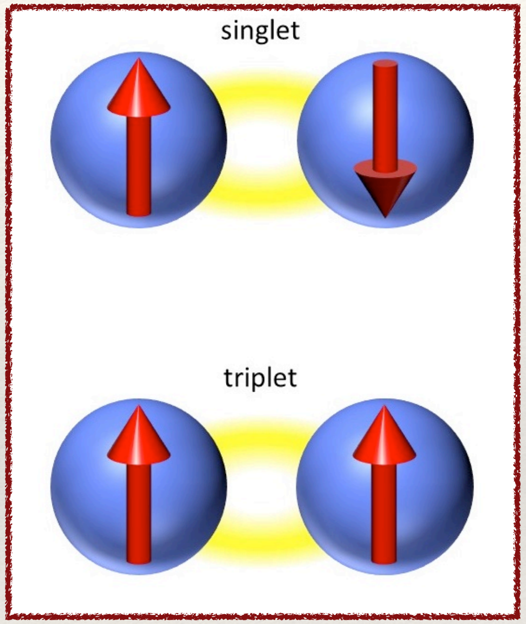




P. Curie demonstrated that a magnet disappears above T_m





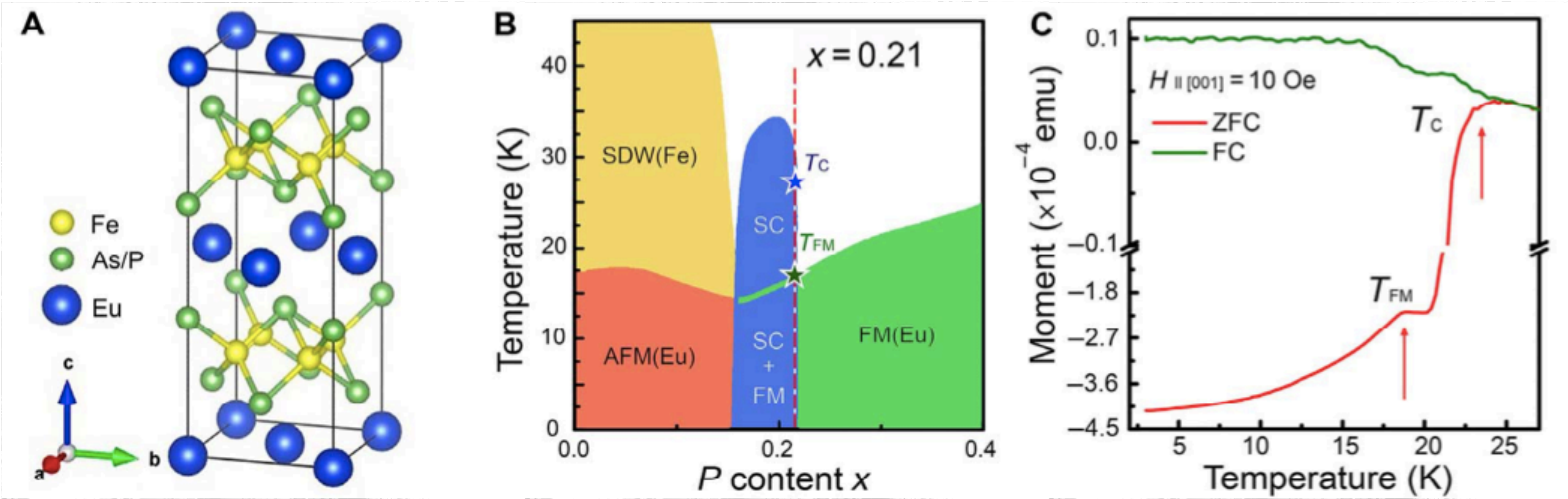


Ferromagnetism and superconductivity are competitors. Why? There are two types of pairing in superconductivity: singlet pairing and triplet one. For the singlet pairing we have two opposite spin projections of the charge carriers in a Cooper pair. For the triplet pairing there are also possibilities of the spins aligned in one direction inside a Cooper pair. As the most of known superconducting materials exhibit the singlet pairing - the ferromagnetism tends to suppress the superconductive order. Triplet pairing is still possible but appears in limited situations.

Coexisting ferromagnetism and superconductivity: The most important is which of the two subsystems is the strongest. In other words, which of the two critical temperatures is the largest- the Curie temperature T_m or the superconductive one T_c

The well-known examples-uranium based compounds UGe_2, URhGe, UCoGe and also $Ho_{1.5}Mo_6S_8$, $ErRh_4B_4$, and $ZrZn_2$. In these materials $T_m \gg T_c$, the singlet pairing is suppressed due to strong exchange interactions, and the triplet superconductivity makes only corrections to a predominantly ferromagnetic state

Recent iron based ferromagnetic superconductors - the new family of ferromagnetic superconductors, where $T_m < T_c$, and the interaction between the superconductive and magnetic subsystems is mediated by the magnetic field (via orbital effects), the weak ferromagnetism does not suppress the singlet pairing



Superconductivity is associated with the Fe-3d electrons, the ferromagnetic ordering is created by the Eu-4f spins. Compounds are possible with a large interval of the P-doping parameter. So that we have $EuFe_2(As_{1-x}P_x)_2$ [while the basic compound is $EuFe_2As_2$]. The ratio T_c/T_m can be varied significantly, the most interesting regime $T_m < T_c$

SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

Domain Meissner state and spontaneous vortex-antivortex generation in the ferromagnetic superconductor $EuFe_2(As_{0.79}P_{0.21})_2$

Vasily S. Stolyarov[1,2,3,4,5]*, Ivan S. Veshchunov[1,6], Sergey Yu. Grebenchuk[1], Denis S. Baranov[1,2,7], Igor A. Golovchanskiy[1,3], Andrey G. Shishkin[1,2], Nan Zhou[8], Zhixiang Shi[8], Xiaofeng Xu[9], Sunseng Pyon[6], Yue Sun[6,10], Wenhe Jiao[11], Guang-Han Cao[11], Lev Ya. Vinnikov[2], Alexander A. Golubov[1,12], Tsuyoshi Tamegai[6], Alexander I. Buzdin[13,14], Dimitri Roditchev[7]*

V. S. Stolyarov et al, Sci. Adv. 2018; 4: eaat1061

Spontaneous pattern formation in pnictides

$$T < T_m < T_c$$

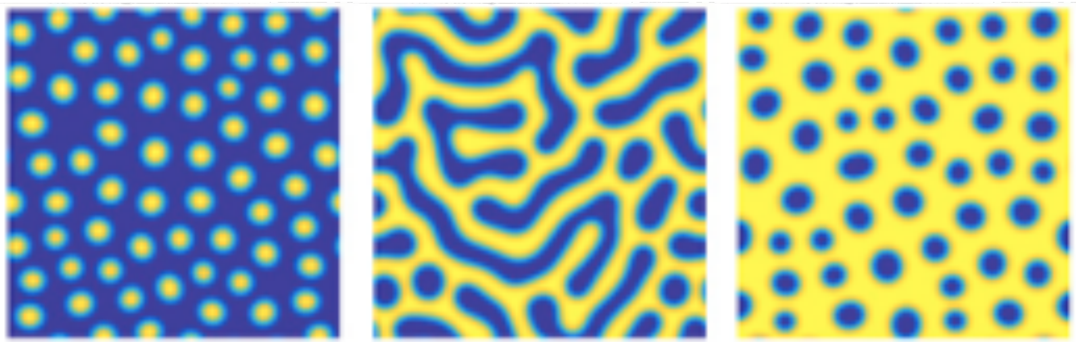
This regime is characterized by active ferromagnetic and superconductive subsystems. When the ferromagnetism is added to superconductivity at $T \lesssim T_m$ and they coexist, the Meissner state becomes spontaneously inhomogeneous, characterized by striped domains (volume effect). At yet lower temperatures (without any external magnetic field), the system generates vortex-antivortex pairs and undergoes a phase transition into a domain vortex-antivortex state characterized by **peculiar Turing patterns**

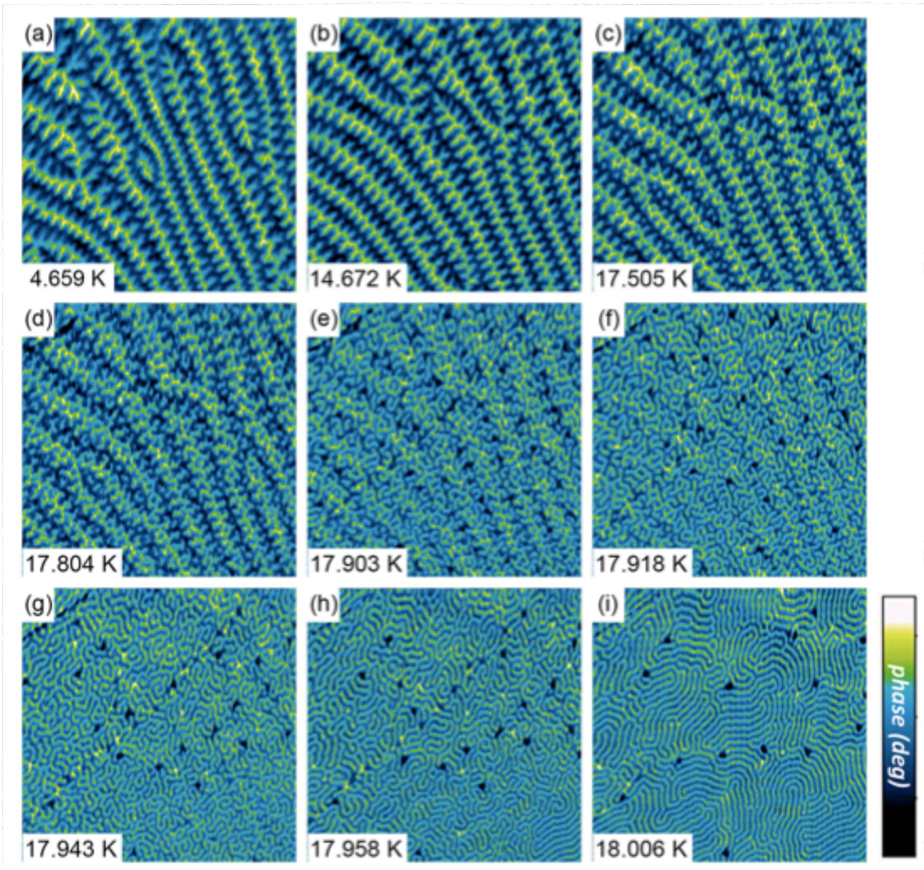
"**The Chemical Basis of Morphogenesis**" is an article written by A. Turing in 1952. It describes how patterns in nature (biology) such as stripes and spirals in the skin of animals and fishes, can arise naturally from a homogeneous, uniform state. In his classic paper, Turing examined the behavior of a system in which two diffusing substances interact with each other, and found that such a system was capable of generating a spatially periodic pattern even from a random or uniform initial state. The theory, which can be called a reaction-diffusion theory of morphogenesis, has become a basic model in theoretical biology. Such patterns are now called Turing patterns. However, they are known not only in biology. This phenomenon is observed in many systems ranging from tissues on planetary surfaces to embryo cells.

A. Turing

Fairy circles are circular patches of land barren of plants, varying between 2 and 12 metres in diameter, often encircled by a ring of stimulated growth of grass. This is also an example of Turing patterns.

A binary mixture of two strains of bacteria that grow on a two-dimensional surface and interact via both short-range contact-dependent killing and long-range growth inhibition (prey-predator model). For the relative densities of the bacteria n_1 and n_2 we have $n_1+n_2=1$, the colour density plots demonstrate the spatial distribution of n_1.



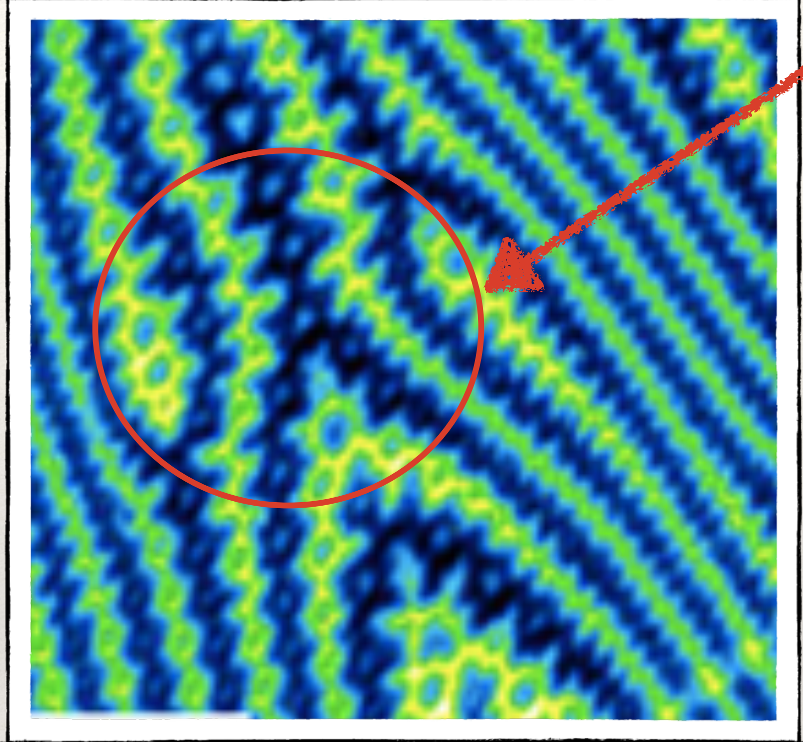


Turing pattern of magnetisation obtained during heating sample $EuFe_2(As_{0.79}P_{0.21})_2$ in zero external magnetic field. Size of scan area is 6x6 μm^2

Turing patterns at the skin of fishes

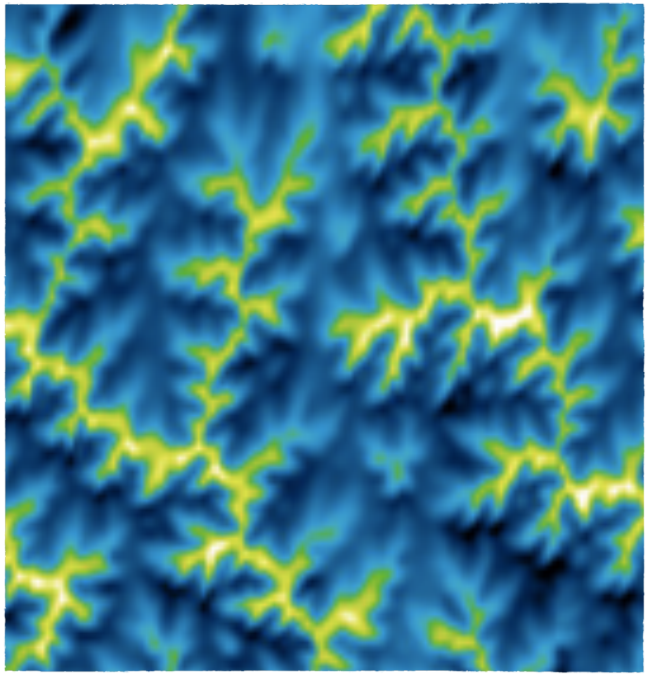
V. S. Stolyarov et al., Sci. Adv. **4**, eaat1061 (2018);
V. S. Stolyarov et al., Phys. Rev. B **98**, 140506(R) (2018)

Eckhaus (fork) instability



Local MFM images of nano-sized spontaneous magnetization patterns in ferromagnetic superconductors in $EuFe_2(As_{0.79}P_{0.21})_2$

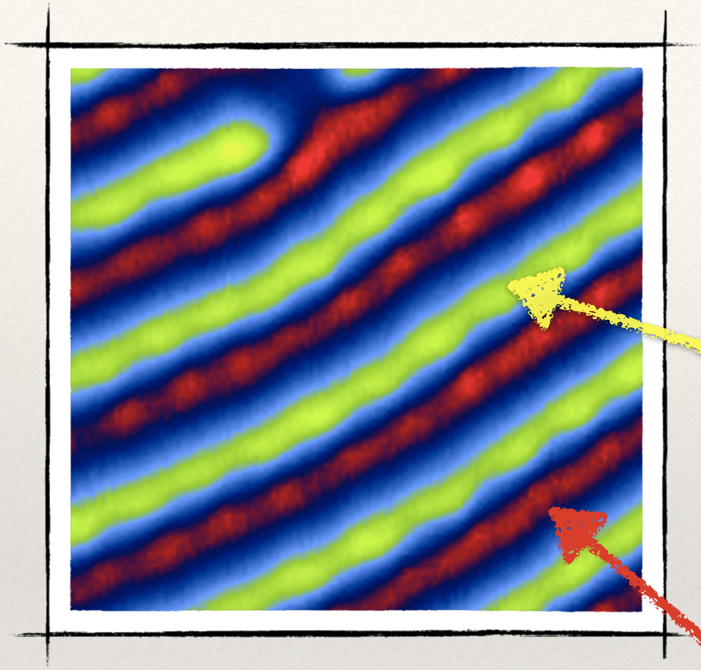
Patterns on the skin of fish



Magnetization spatial distribution (MFM) in $EuFe_2(As_{0.79}P_{0.21})_2$: patterns of the type “ice on glass”

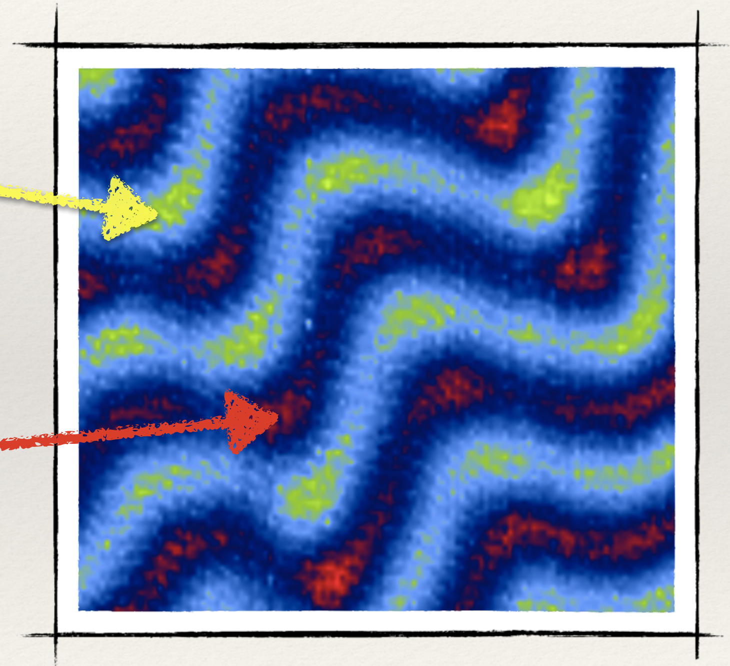
Real ice patterns on glass

Magnetization spatial distribution (MFM) in $EuFe_2(As_{0.79}P_{0.21})_2$: stripes of magnetisation accompanied by the vortex-antivortex patterns of the superconducting condensate



Antivortices

Vortices



V. S. Stolyarov et al, Sci. Adv. **4**, eaat1061 (2018);
V. S. Stolyarov et al, Phys. Rev. B **98**, 140506(R) (2018)

$$T_m < T < T_c$$

This regime is much less studied so far. Previously it was considered that this regime is not of interest because the magnetic system is passive here (paramagnetic regime). However, new results have demonstrated that the magnetic response of the system is also unusual for such temperatures. It is known since long ago that in this temperature interval the system undergoes the crossover from superconductivity type II to type I.

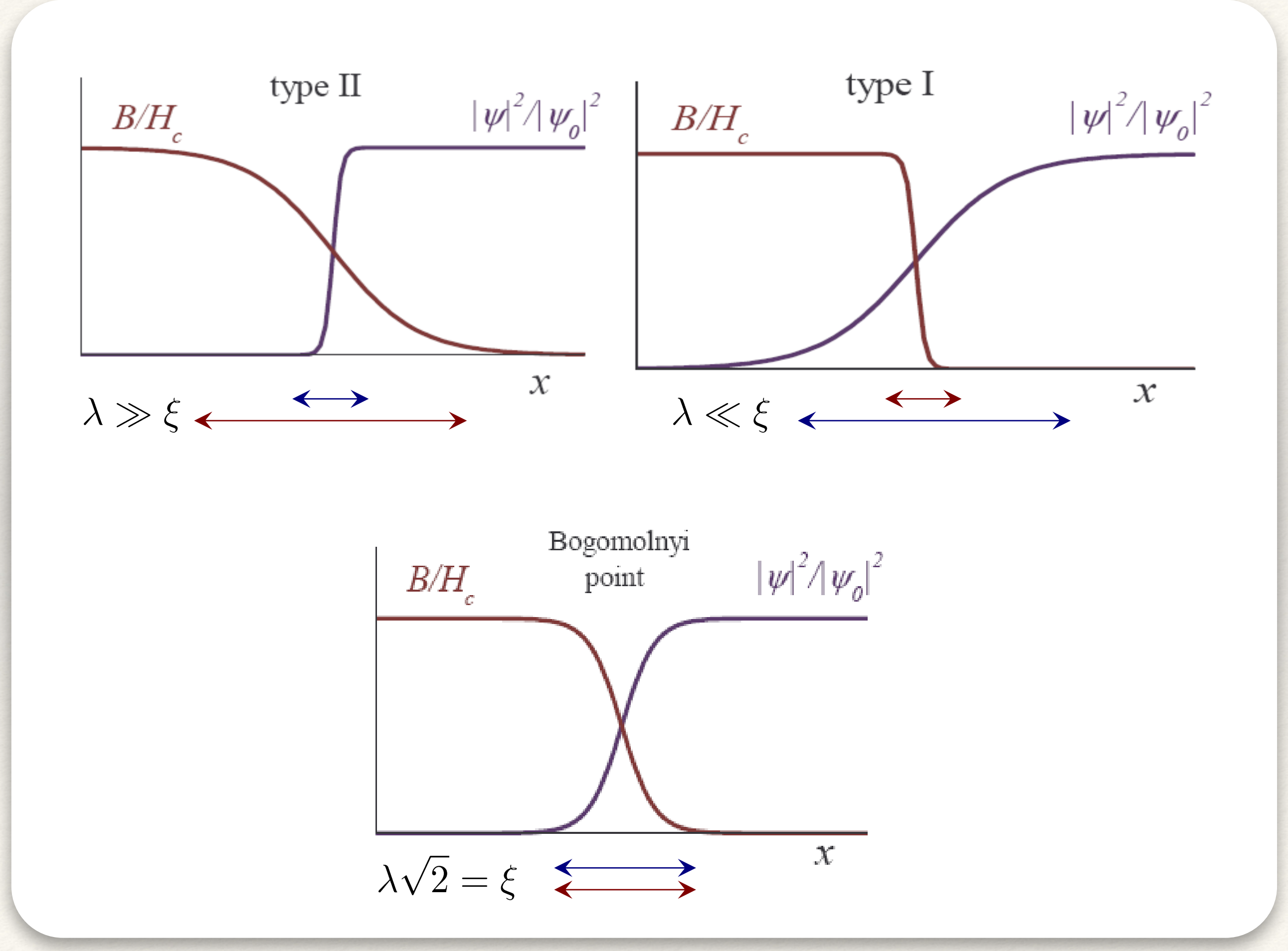
L. N. Bulaevskii, A. I. Buzdin, M. L. Kulic, and S. V. Panjukov, Advances in Physics **34**, 175 (1985)

There is a temperature window, situated between T_m and T_c, where the intrinsically type-II superconductors are in the intertype regime with its rich phase diagram characterized by exotic spatial flux configurations - vortex clusters, chains, giant vortices and vortex liquid droplets.

A. Vagov, T. T. Saraiva, A. A. S., A. S. Vasenko, J. A. Aguiar, V. S. Stolyarov, and D. Roditchev, Communications Physics (accepted)

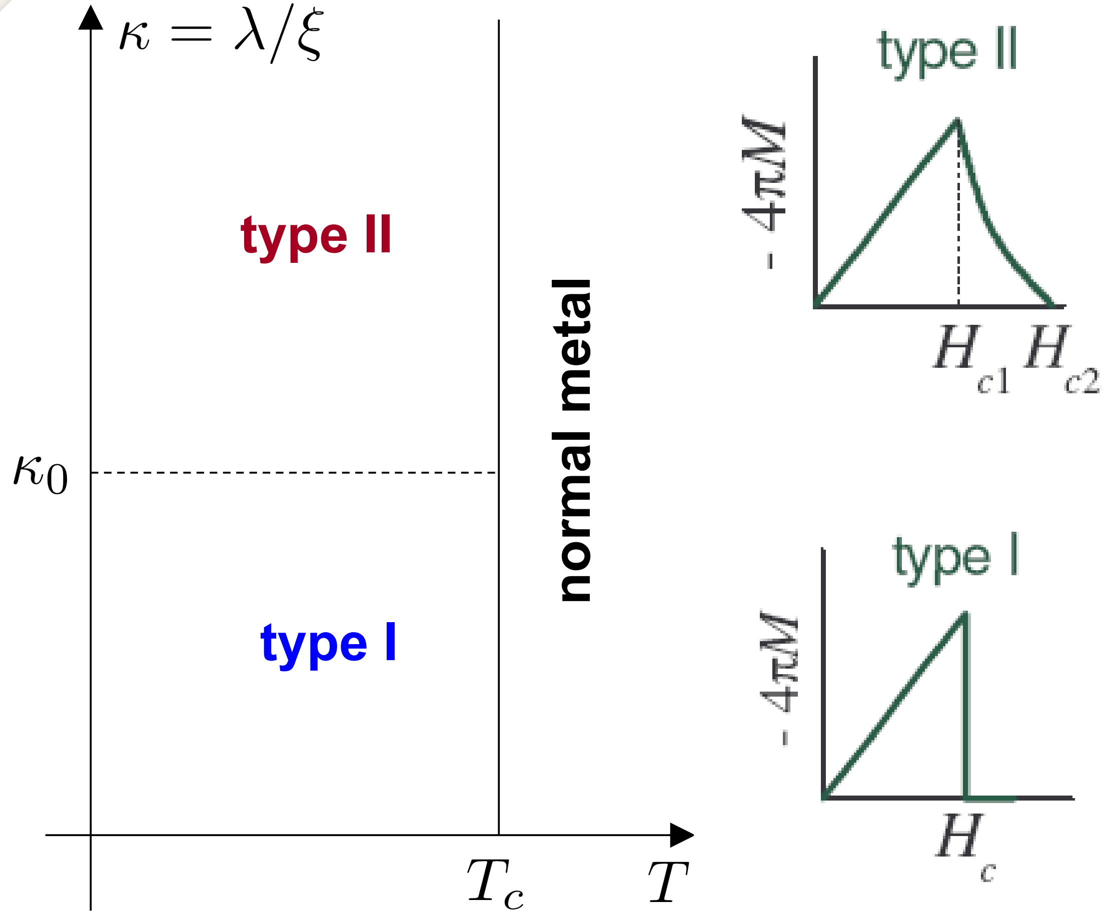
Intertype (IT) physics and superconductors

Ginzburg-Landau theory



Phase digram of the magnetic response

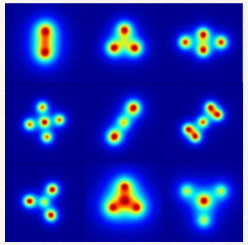
Ginzburg-Landau theory

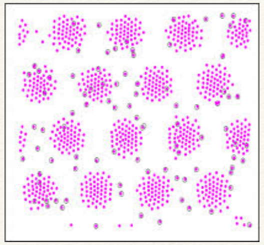


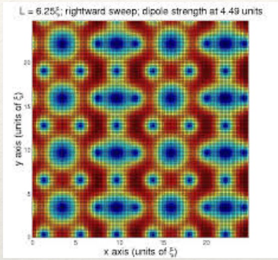
H_c, H_{c1}, H_{c2}

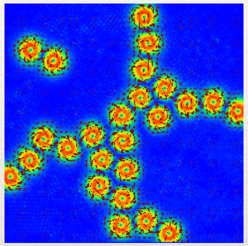
$\kappa_0 = 1/\sqrt{2}$

Bogomolnyi point degeneracy

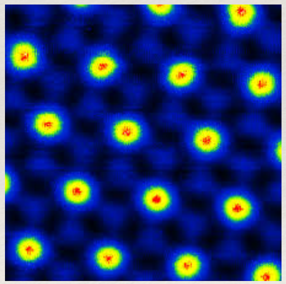








Ginzburg-Landau theory



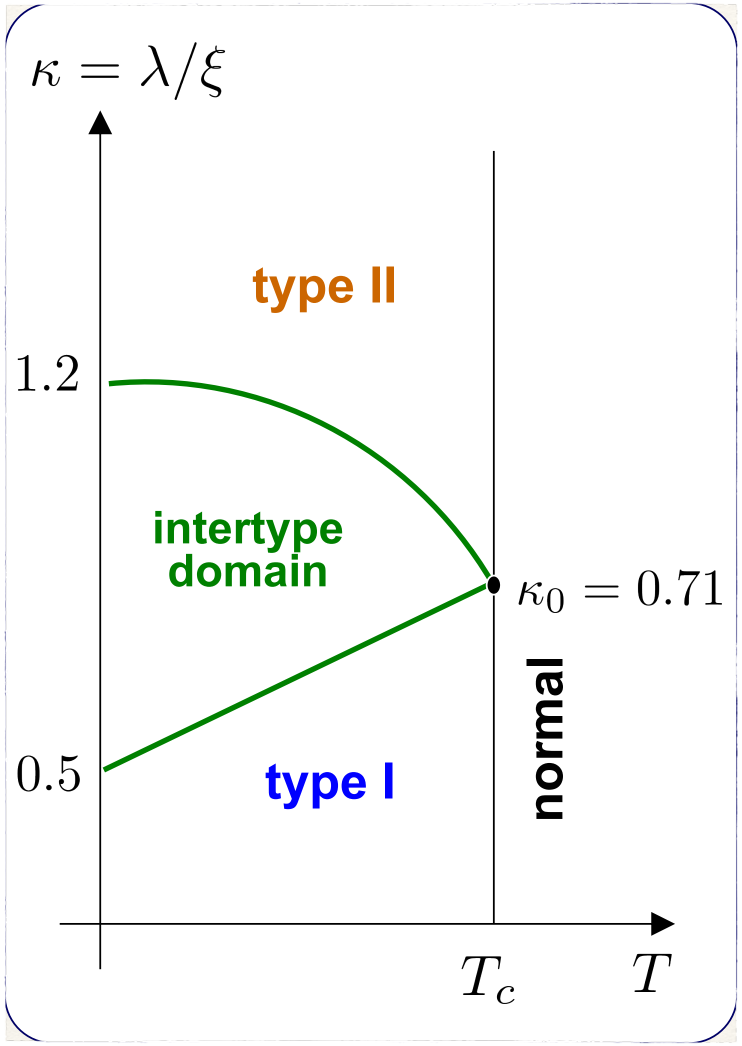
type I

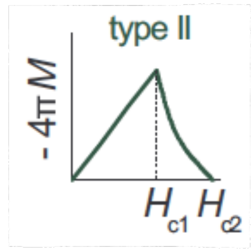
$\kappa = \kappa_0 = 1/\sqrt{2}$

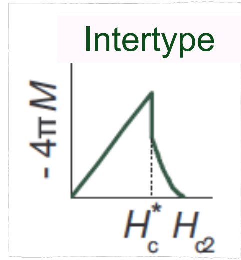
type II

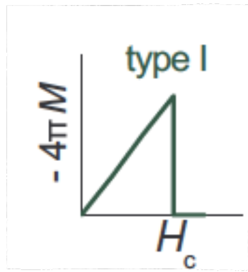
However, the Ginzburg-Landau theory gives an adequate picture of the types of superconductivity (for conventional superconductors with singlet pairing) only in the limit $T \to T_c$. As the temperature decreases, a finite intertype domain appears between types I and II on the $\kappa - T$ plane.

J. Auer and H. Ullmaier, Phys. Rev. B **7**, 136 (1973)
A. E. *Jacobs, Phys. Rev. Lett.* ***26****, 629 (1971)*
M. Laver et al., Phys. Rev. B **79**, 014518 (2009)
J.-Y. Ge et al., Phys. Rev. B **90**, 629184511 (2014)
T. Riemann et al., Nature Comm. **6**, 8813 (2015)
A. Vagov et al., Comm. Physics **3**, 58 (2020)

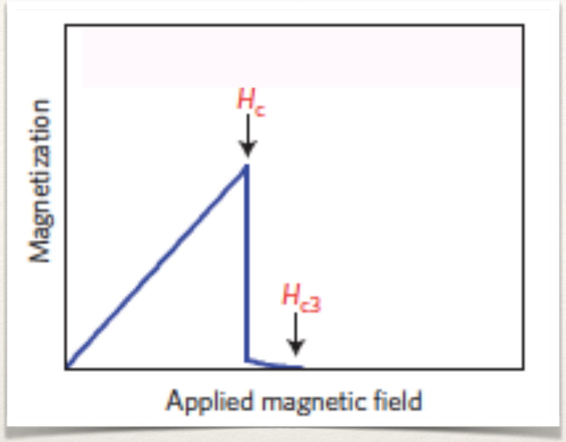




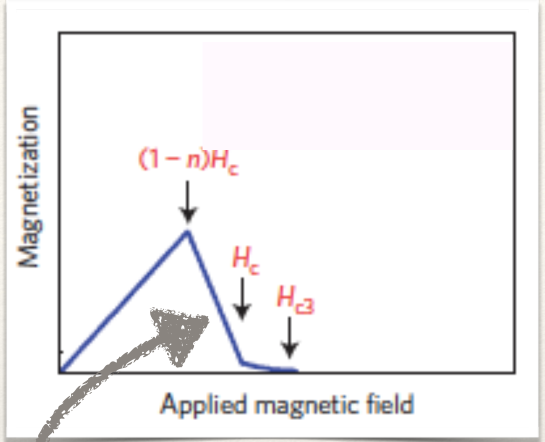




Intermediate mixed state Nb (experiment)

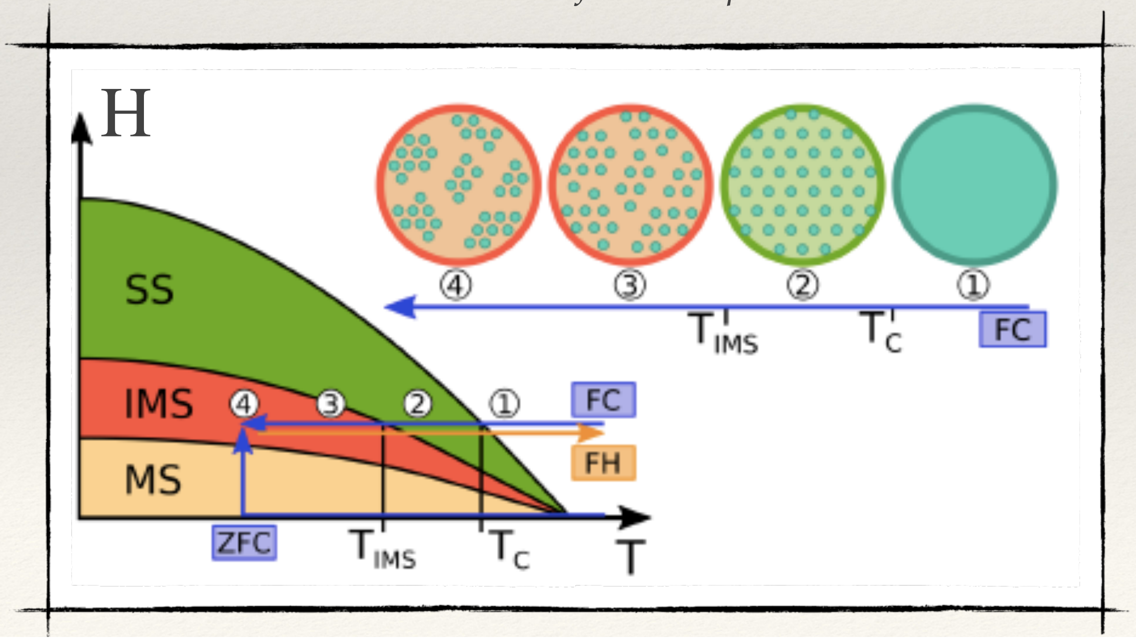


Infinite sample



Finite sample

IMS



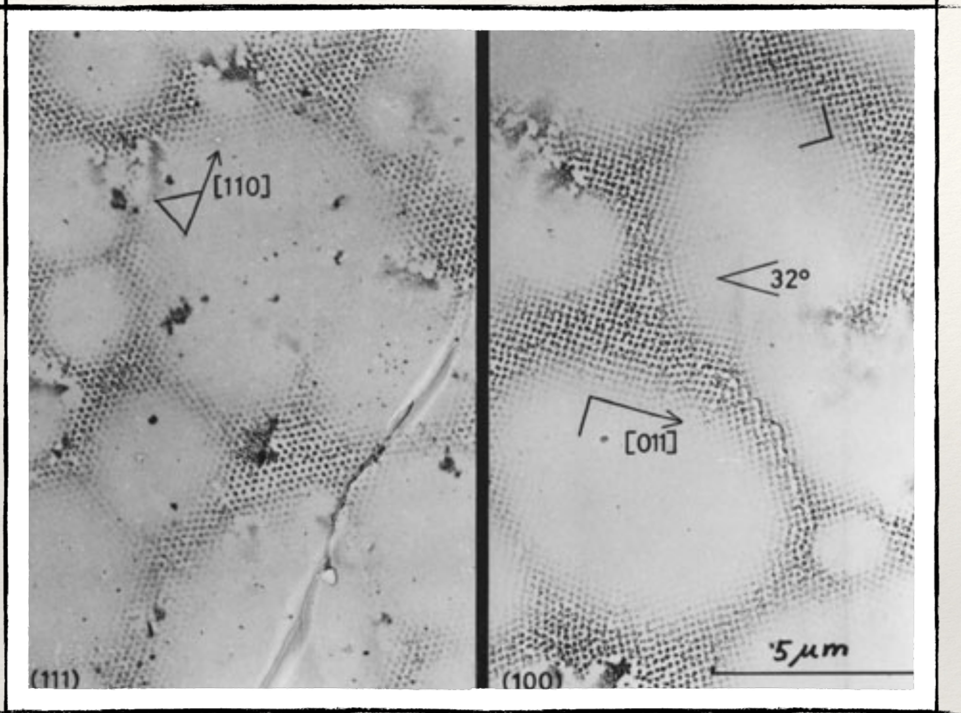
MS - Meissner state

IMS - Intermediate mixed state

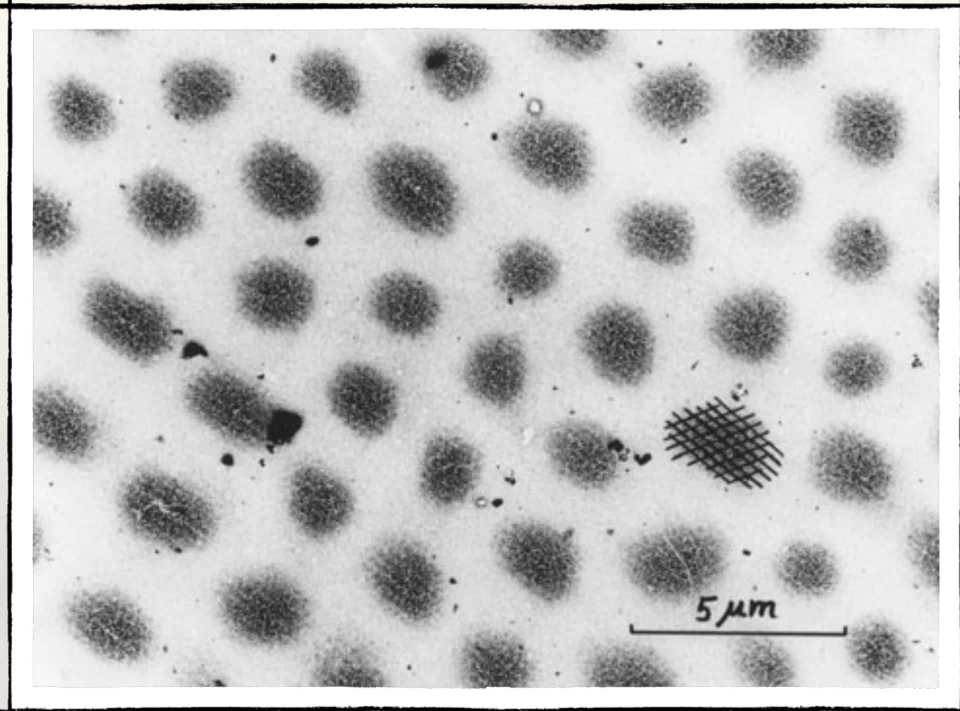
SS - Shubnikov state (Abrikosov lattice)

A. Backs et al., Phys. Rev. B **100**, 064503 (2019)

Intermediate mixed state Nb (experiment)



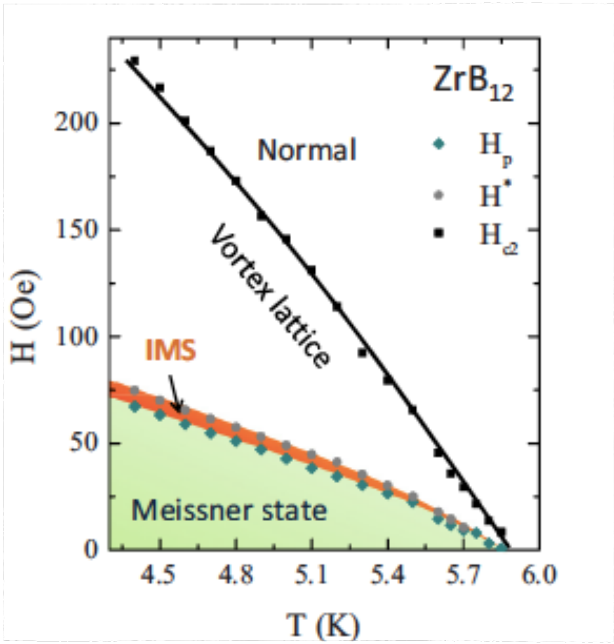
Meissner islands embedded in the vortex matter

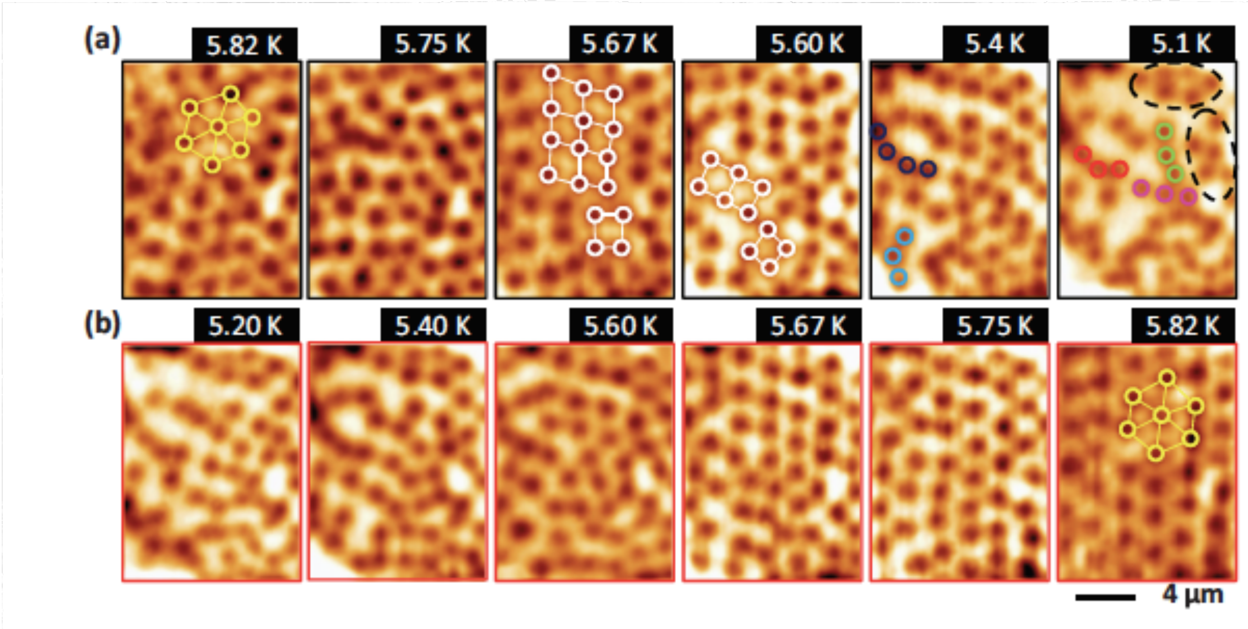


Vortex islands embedded in the Meissner matrix

H. E. Brandt and M. Das, J Supercond Nov Magn **24**, 7 (2011)

Intermediate mixed state ZrB_{12} (experiment)

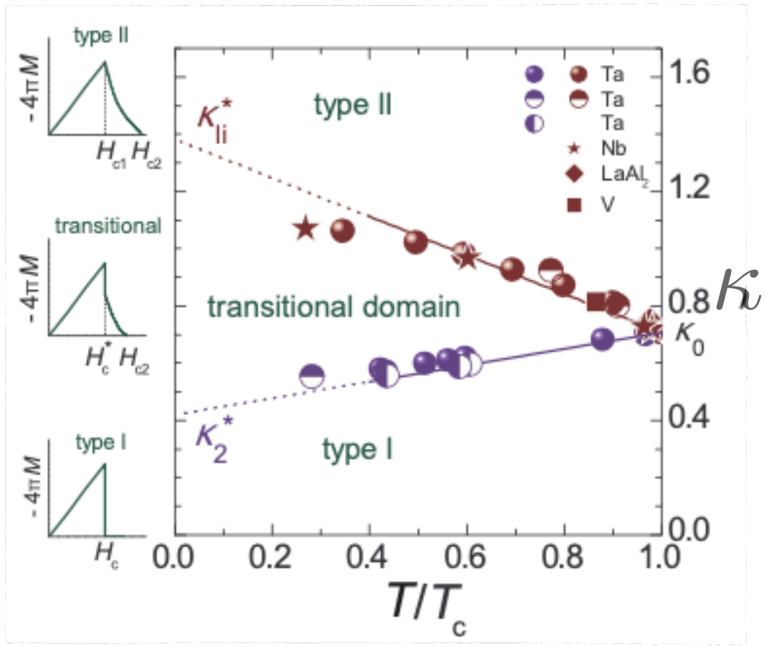




Field Cooling at H=4.73 Oe [Scanning Hall probe microscopy]

J.-*Y. Ge et al.*, Phys. Rev. B **90**, 184511 (2019)

Intermediate mixed state (theory and comparison with experiment)



Intertype (transitional) domain between types I and II in the phase diagram of the superconductive magnetic response

A. Vagov, A. A. Shanenko, V. V. Milosevic,V. M. Axt, V. M. Vinokur, J. Albino Aguiar, and F. M. Peeters, Phys. Rev. B **93**, 174503 (2016)

A. Vagov, S. Wolf, M. D. Croitoru, and A. A. Shanenko, Comm. Phys. **3**, 58 (2020).

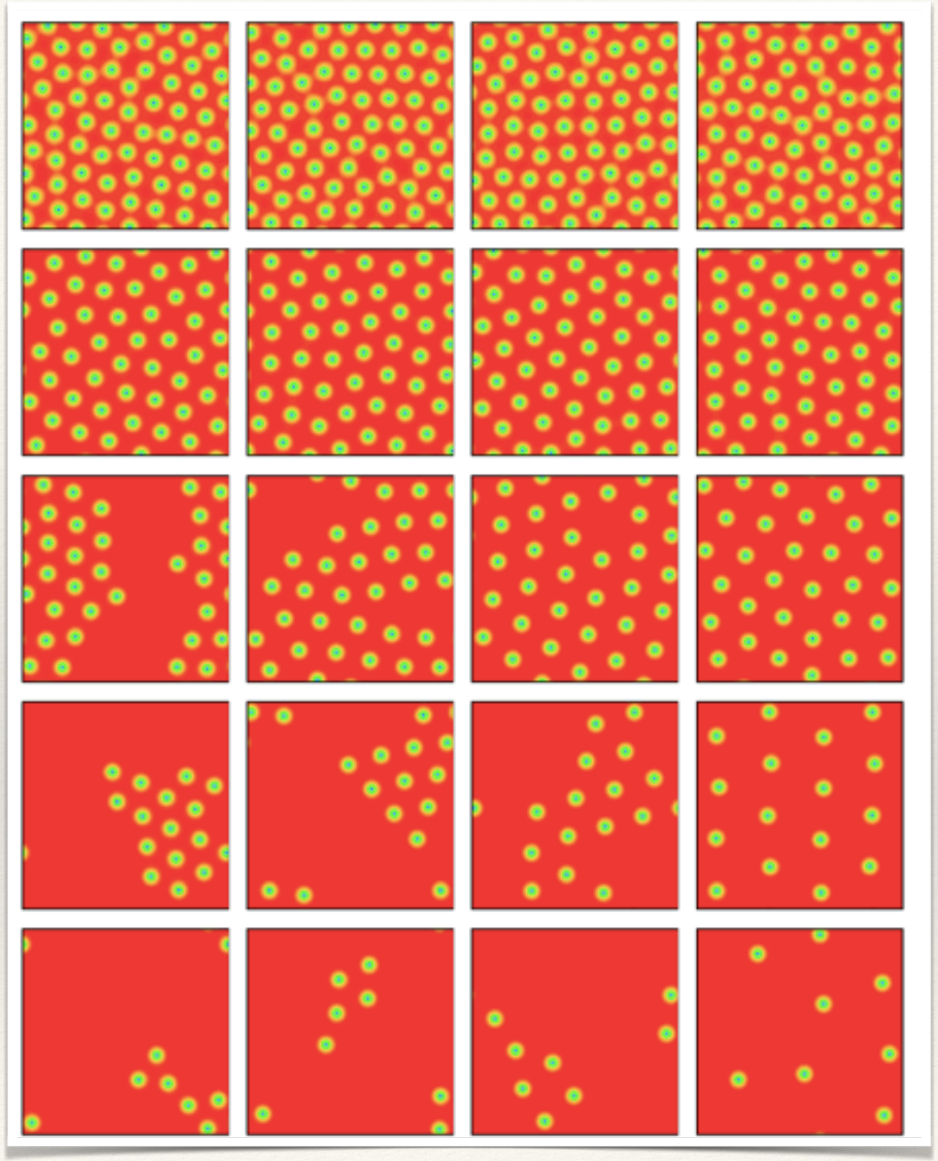
How to find the boundaries of the intertype domain in the phase diagram (κ-T plane)? First, we calculate the Gibbs free energy difference between the Meissner state and an arbitrary nonuniform condensate solution at the thermodynamic critical field H_c. Then, we consider a nearly disappearing spatially inhomogeneous condensate and investigate when its Gibbs free energy is equal to that of the Meissner state. This assumes that $H_c = H_{c2}$. Second, we calculate the Gibbs free energy for the two-vortex solution (again at H_c) as a function of the distance between these vortices, extract the long-range asymptote for large vortex separations and find when this long-range asymptote changes its sign.

In the standard GL approach both these conditions give the same boundary between types I and II - namely, $\kappa = \kappa_0 = 1/\sqrt{2}$. Beyond the GL approach these conditions produce the upper ($H_c = H_{c2}$) and lower (long-range attraction between vortices) temperature dependent boundaries of the intertype domain in the κ-T plane. For example, within the extended GL formalism (only leading corrections to the GL theory, the clean limit and spherical Fermi surface) one obtains (here $\tau = 1 - T/T_c$)

$$\kappa_u = \kappa_0(1 + 0.95\tau)$$
$$\kappa_l = \kappa_0(1 - 0.407\tau)$$

Universal relations in agreement with experiments

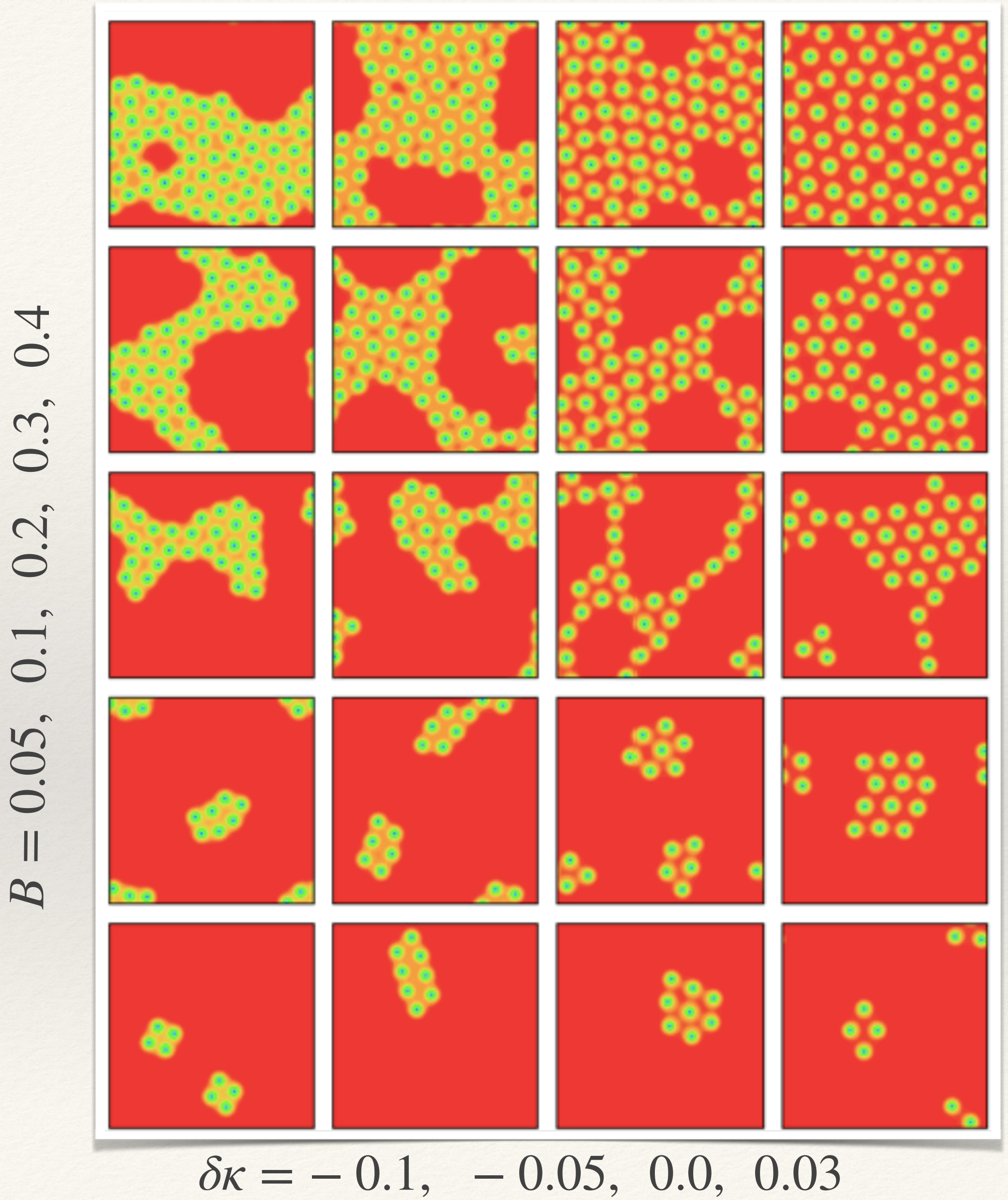
IMS - Intermediate mixed state



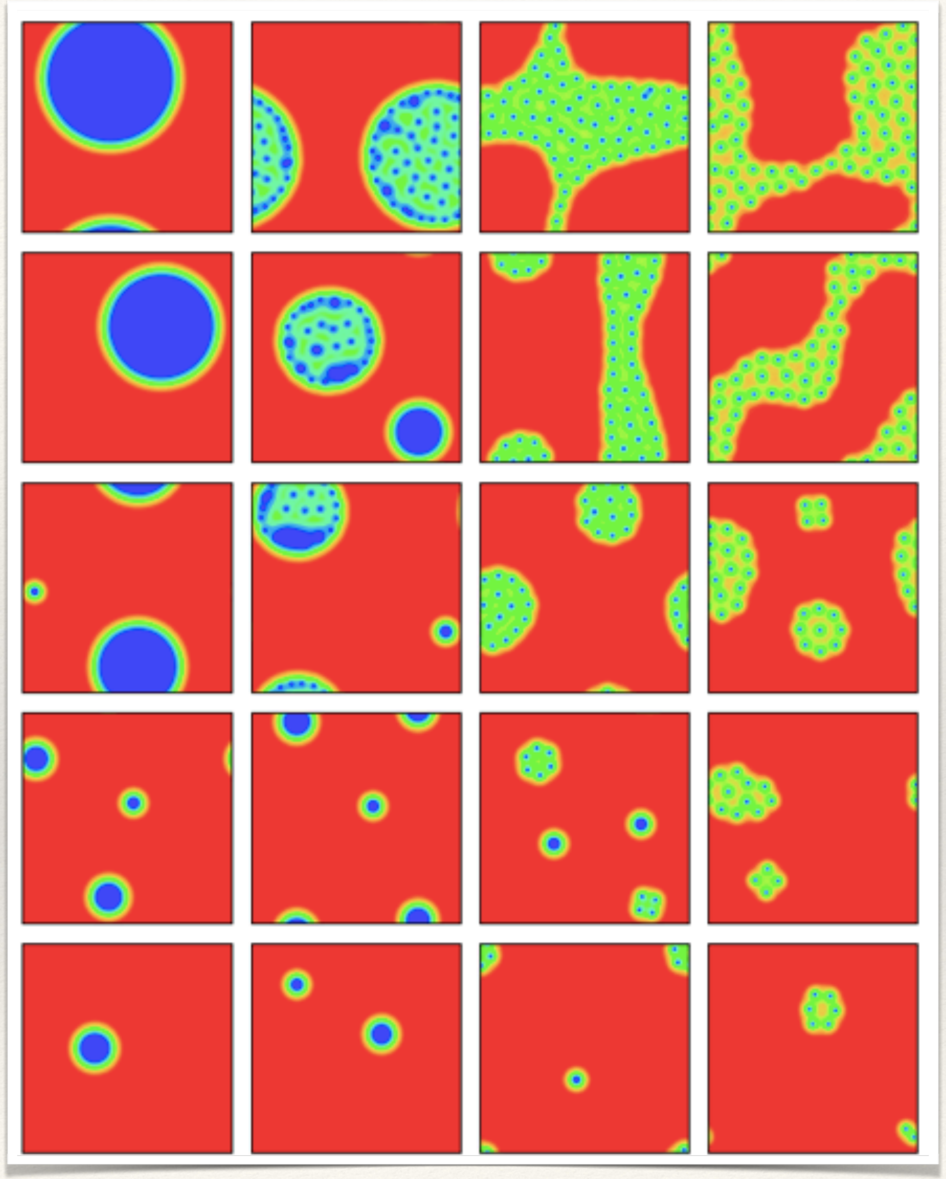
$B = 0.05,\ 0.1,\ 0.2,\ 0.31$

$\delta\kappa = 0.08,\ 0.1,\ 0.13,\ 0.3$

Vortices in IMS, closer to type II, *A. Vagov et al.*, Communications Physics **3**, 58 (2020) [$\delta\kappa = \kappa - \kappa_0$]



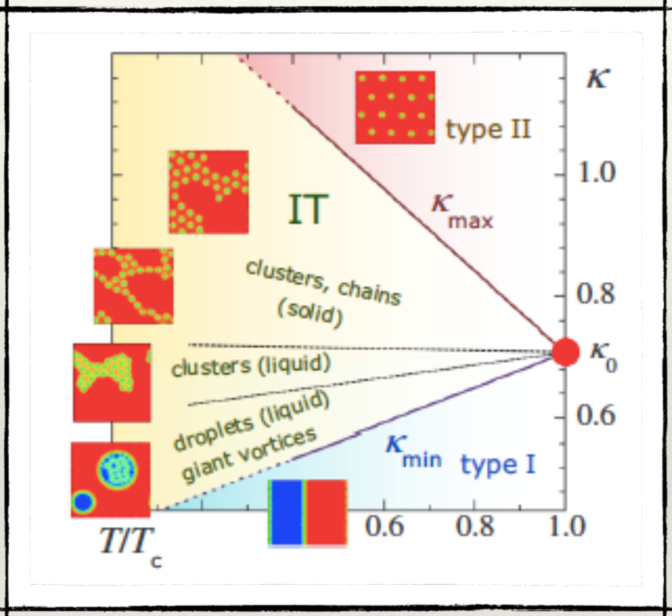
Vortices in IMS, closer to type I, *A. Vagov et al.*, Communications Physics **3**, 58 (2020) [$\delta\kappa = \kappa - \kappa_0$].



$B = 0.05,\ 0.1,\ 0.2,\ 0.3,\ 0.4$

$\delta\kappa = -0.28,\quad -0.25,\quad -0.2,\quad -0.15$

Vortices in IMS, in the nearest vicinity of type I, *A. Vagov et al.*, Communications Physics **3**, 58 (2020) [$\delta\kappa = \kappa - \kappa_0$].



Different variants of the distribution of vortices the IMS (intermediate mixed state) in superconductors between types I and II. The region on the phase diagram between types I and II is the region of intertype behavior. Closer to type I, patterns are observed within which there is a lattice distribution of vortices. Closer to type II, there is a qualitative change in the structure of the patterns from solid to liquid. Finally, in the nearest vicinity of type I giant vortices begin to predominate, tending to the formation of lamellas.

Minimal Theoretical Model for $T_m < T < T_c$

The free energy density of this model has three components (superconductive, magnetic, and interaction between them)

$$f = f_s + f_m + f_{\text{int}}$$

The free energy density of the (ferro)magnetic subsystem (the GL free energy density)

$$f_m = \frac{a_m}{2}\mathbf{M}^2 + \frac{b_m}{4}(\mathbf{M}^2)^2 + \frac{\mathcal{K}_m}{2}\sum_i (\nabla_i \mathbf{M})^2$$

The free energy density of the interaction between two subsystems

$$f_{\text{int}} = \gamma \mathbf{M}^2 |\Delta|^2 - \mathbf{M} \cdot \mathbf{B}$$

Here $\mathbf{M}$ is the magnetization vector and $a_m, b_m, \mathcal{K}_m$, and γ are the relevant parameters for the system, we assume $a_m = \alpha_m (T - \theta)$, where θ is the Curie point of the uncoupled magnetic subsystem.

The free energy density for the superconductive subsystem is taken within the extended GL model as $T_m < T_c$, i.e.

$$f_s = \frac{\mathbf{B}^2}{8\pi} + (\mathrm{g}^{-1} - a_1)|\Delta|^2 + a_2|\mathbf{D}\Delta|^2 - a_3\Big(|\mathbf{D}^2\Delta|^2 + \frac{1}{3}\mathrm{rot}\mathbf{B}\cdot\mathbf{i} + \frac{4e^2}{\hbar^2\mathbb{c}^2}\mathbf{B}^2|\Delta|^2\Big) + \frac{b_1}{2}|\Delta|^4 - \frac{b_2}{2}\Big[8|\Delta|^2|\mathbf{D}\Delta|^2 + \Delta^{*2}(\mathbf{D}\Delta)^2 + \Delta^2(\mathbf{D}^*\Delta^*)^2\Big] - \frac{c_1}{3}|\Delta|^6 + \dots$$

Where Δ is the order parameter (s-wave single-band pairing),

$$\mathbf{D} = \boldsymbol{\nabla} - \mathbb{i}2e\mathbf{A}/\hbar\mathbb{c}, \ \mathbf{i} = 4e\mathrm{Im}[\Delta^*\mathbf{D}\Delta]/\hbar\mathbb{c}$$

and the coefficients are given by (we consider the clean limit)

$$a_1 = N(0)\ln\left(\frac{2e^\Gamma\hbar\omega_D}{\pi T}\right), \ b_1 = N(0)\frac{7\zeta(3)}{8\pi^2T^2}, c_1 = N(0)\,\frac{93\zeta(5)}{128\,\pi^4T^4},$$
$$a_2 = \frac{b_1}{6}\hbar^2v_F^2, \ a_3 = \frac{c_1}{30}\hbar^4v_F^4, \ b_2 = \frac{c_1}{9}\hbar^2v_F^2.$$

Here ω_D is the Debye (cutoff) frequency, $N(0) = mk_F/2\pi^2\hbar^2$ is the DOS at the Fermi surface, k_F and $v_F = \hbar k_F/m$ are the Fermi wavenumber and velocity, and $\zeta(\ldots)$ is the Riemann zeta function.

To simplify our consideration, we employ the perturbative approach. A small parameter for the perturbation expansion is the proximity to the superconductive critical temperature $\tau = 1 - T/T_c$. The calculation is done for the temperature interval $T_m < T < T_c$, so that the magnetic order parameter $\mathbf{M}$ is zero when the coupling between the subsystems is absent (the paramagnetic regime).

The solution to the pertinent physical quantities is sought in the form of the following series expansions:

$$\Delta = \tau^{1/2}\Psi + \tau^{3/2}\psi + \ldots, \; \mathbf{M} = \tau\boldsymbol{\mathcal{M}} + \tau^2\mathfrak{m} + \ldots,$$
$$\mathbf{B} = \tau\boldsymbol{\mathcal{B}} + \tau^2\mathfrak{b} + \ldots, \; \mathbf{A} = \tau^{1/2}\boldsymbol{\mathcal{A}} + \tau^{3/2}\mathfrak{a} + \ldots.$$

Furthermore, we take into account that in the vicinity of T_c, the superconductor characteristic lengths are divergent. Introducing the spatial scaling $\mathbf{r} \to \tau^{1/2}\mathbf{r}$, one obtains the scaling factor for the spatial gradients as $\nabla \to \tau^{-1/2}\nabla$.

In the τ-expansion of the free energy the order $\mathcal{O}(\tau)$ disappears due to the equation for T_c. The order $\mathcal{O}(\tau^2)$ yields the superconductor GL theory modified by the linear coupling to the magnetic subsystem. The corresponding GL equations read as

$$\left(a + b|\Psi|^2\right)\Psi - \mathcal{K}\boldsymbol{\mathcal{D}}^2\Psi = 0 \quad \left(\boldsymbol{\mathcal{D}} = \boldsymbol{\nabla} - \mathbb{i}\frac{2e}{\hbar\mathbb{c}}\boldsymbol{\mathcal{A}}\right),$$

$$\mathrm{rot}\left[\boldsymbol{\mathcal{B}} - 4\pi\boldsymbol{\mathcal{M}}\right] = \frac{4\pi}{\mathbb{c}}\mathbf{j} \qquad \left(\mathbf{j} = \frac{4e\mathcal{K}}{\hbar}\mathrm{Im}[\Psi^*\boldsymbol{\mathcal{D}}\Psi]\right),$$

$$a_m\boldsymbol{\mathcal{M}} = \boldsymbol{\mathcal{B}},$$

where the coefficients a, b, c, and $\mathcal{K}$ are obtained from the τ-expansion of the previous temperature-dependent coefficients in the free energy of the superconductive system as

$$a = -N(0), \quad b = N(0)\frac{7\zeta(3)}{8\pi^2T_c^2}, \quad c = N(0)\,\frac{93\zeta(5)}{128\pi^4T_c^4}, \quad \mathcal{K} = \frac{b}{6}\hbar^2v_F^2.$$

For the stationary point the free energy density can be written in the form

$$f^{(0)} = \frac{\boldsymbol{\mathcal{B}}^2}{8\pi\mu} + \mathcal{K}\left|\boldsymbol{\mathcal{D}}\Psi\right|^2 + a|\Psi|^2 + \frac{b}{2}|\Psi|^4, \; \mu = \left(1 - \frac{4\pi}{a_m}\right)^{-1}.$$

↓

A superconductor placed in a magnetic medium with the magnetic permeability μ.

Now, we introduce the dimensionless units

$$\tilde{\mathbf{r}} = \frac{\mathbf{r}}{\lambda_\mu\sqrt{2}}, \; \tilde{\boldsymbol{\mathcal{B}}} = \frac{\kappa_\mu\sqrt{2}}{\mu H_c^{(0)}}\boldsymbol{\mathcal{B}}, \; \tilde{\boldsymbol{\mathcal{A}}} = \frac{\kappa_\mu}{\mu H_c^{(0)}\lambda_\mu}\boldsymbol{\mathcal{A}}, \; \tilde{\Psi} = \frac{\Psi}{\Psi_0}, \; \tilde{f} = \frac{4\pi f}{\mu H_c^{(0)2}},$$

where $\Psi_0 = \sqrt{-a/b}$, $H_c^{(0)}$ is the thermodynamic critical field in the GL domain $H_c^{(0)} = \sqrt{4\pi a^2/b\mu}$, and the effective magnetic penetration depth λ_μ and the effective GL parameter κ_μ are given by

$$\lambda_\mu = \frac{\lambda}{\sqrt{\mu}}, \quad \kappa_\mu = \frac{\kappa}{\sqrt{\mu}},$$

where λ and κ are the magnetic penetration depth and the GL parameter of the decoupled superconductive subsystem. In dimensionless units the GL free energy density writes

$$f^{(0)} = \frac{\boldsymbol{\mathcal{B}}^2}{4\kappa_\mu^2} + \frac{1}{2\kappa_\mu^2}|\boldsymbol{\mathcal{D}}\Psi|^2 - |\Psi|^2 + \frac{1}{2}|\Psi|^4, \quad \boldsymbol{\mathcal{D}} = \boldsymbol{\nabla} + \mathbb{i}\boldsymbol{\mathcal{A}}.$$

The GL system is governed by the effective GL parameter κ_μ and the boundary between types I and II is given by ($\kappa_\mu^* = \kappa_0$ or $\kappa^*/\sqrt{\mu} = \kappa_0$)

$$\kappa^* = \kappa_0\sqrt{\frac{T-\theta}{T-T_m}}, \; a_m(T_m) = 4\pi.$$

When we go beyond the GL theory, including the leading order correction in τ, we find for the boundaries of the intertype domain

$$\kappa^* = \kappa_0\sqrt{\mu}\left\{1+\tau\left[1-\bar{c}+2\bar{\mathcal{Q}}+\bar{\gamma}+\left(2\bar{\mathcal{L}}-\bar{c}-\frac{5\bar{\mathcal{Q}}}{3}-\bar{\gamma}+\mu\bar{\mathcal{K}}_m\right)\frac{\mathcal{J}}{\mathcal{I}}\right]\right\}$$

where

$$\mathcal{I} = \int|\Psi|^2\left(1-|\Psi|^2\right)d^2\mathbf{r},\ \mathcal{J} = \int|\Psi|^4\left(1-|\Psi|^2\right)d^2\mathbf{r},$$

and

$$\bar{c} = \frac{ca}{3b^2},\ \bar{\mathcal{Q}} = \frac{\mathcal{Q}a}{\mathcal{K}^2},\ \bar{\mathcal{L}} = \frac{\mathcal{L}a}{b\mathcal{K}},\ \bar{\gamma} = -\frac{\gamma a\Phi_0^2}{2a_m^2\mathcal{K}^2},\ \bar{\mathcal{K}}_m = -\frac{2\pi\mathcal{K}_m a}{a_m^2\mathcal{K}}.$$

As the coefficients c, $\mathcal{Q}$, and $\mathcal{L}$ are obtained as

$$c = N(0)\,\frac{93\zeta(5)}{128\,\pi^4 T_c^4},\ \mathcal{Q} = \frac{c}{30}\hbar^4 v_F^4, \mathcal{L} = \frac{c}{9}\hbar^2 v_F^2.$$

Ferromagnetic-superconducting pnictides as IT superconductors

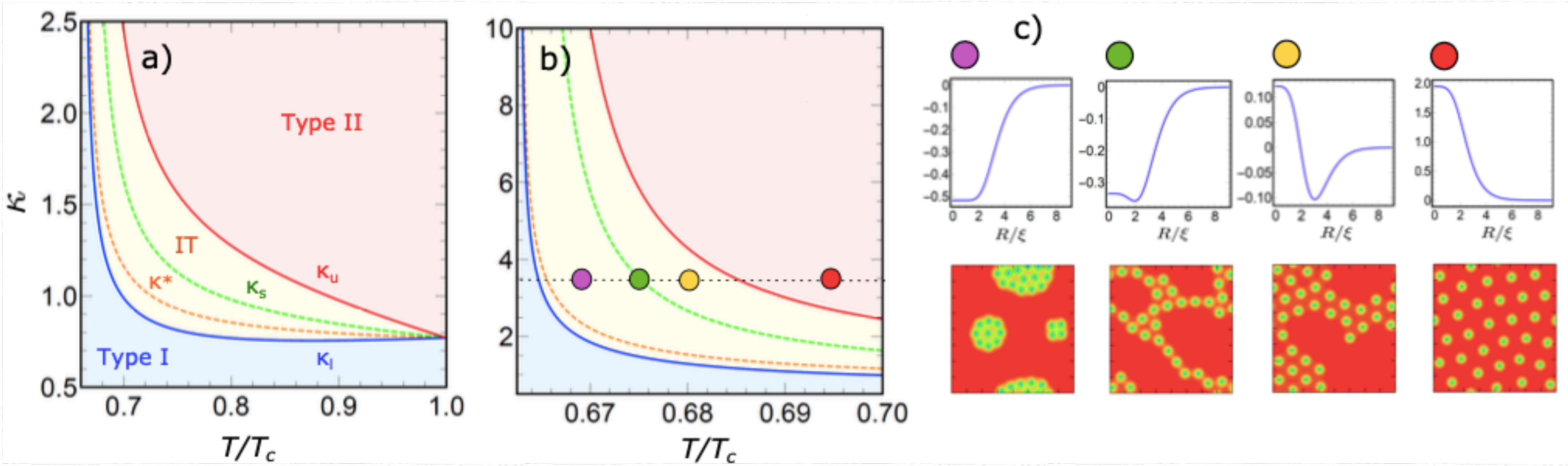
The boundary between IT and type II (the upper boundary of the IT domain) is defined by the onset of the long-range vortex-vortex attraction, which makes the mixed state with a vortex lattice unstable at low magnetic fields. The vortex-vortex interaction potential is calculated from the Gibbs free energy using the two-vortex solution of the GL equations and keeping only the contribution depending on the distance between vortices. Then, changing the sign of the long-range interaction potential is obtained from the condition of the zero Gibbs free energy when using the asymptote of the two-vortex solution of the GL equations at large distance R between vortices. As a result, one gets $J(R \to \infty) = 2I(R \to \infty)$. Using the solution of the GL formalism, we find the upper boundary as

$$\kappa_u = \kappa_0\sqrt{\mu}\left[1 + \tau\left(1 - 3\bar{c} + 4\bar{\mathcal{L}} - \frac{4\bar{\mathcal{Q}}}{3} - \bar{\gamma} + 2\mu\bar{\mathcal{K}}_m\right)\right]$$

The boundary between the IT and type-I superconductivity (the lower boundary of the IT domain) is obtained from the condition of the IMS appearance/disappearance expressed by the equality of the thermodynamic and upper critical fields. It is equivalent to the condition of the zero Gibbs free energy difference between a non-homogeneous nearly disappearing solution $\Psi \to 0$ and the Meissner state at the thermodynamic critical field (a spatially nonuniform solution is more favourable than the Meissner uniform solution). As $\Psi \to 0$, one obtains $J \ll I$, or $J/I = 0$. This yields the lower boundary of the IT domain as

$$\kappa_l = \kappa_0 \sqrt{\mu}\left[1 + \tau\left(1 - \bar{c} + 2\bar{\mathcal{Q}} + \bar{\gamma}\right)\right]$$

For the clean system with the spherical Fermi surface we find the universal constants $\bar{c} = -0.227$, $\bar{\mathcal{L}} = -0.454$, $\bar{\mathcal{Q}} = -0.817$. The coefficients related to the magnetic subsystem are not given by universal constants and depend on the microscopic characteristics of both the superconducting and magnetic subsystems. However, our qualitative results are general and not sensitive to a particular choice of these coefficients. For illustration we choose the dimensionless parameters as $\bar{\gamma} = 0, \bar{\mathcal{K}}_m = 1$.



Panels (a) and (b) show the upper κ_u and lower κ_l boundaries of the intertype domain. κ^* separates types I and II in the GL theory, κ_s is the line of the zero surface tension. Panel (c) illustrates the pairwise vortex interaction potential with the corresponding vortex configurations at low magnetic fields (red - Meissner domains, green - vortices) for different points on the phase diagram.

Conclusions

- Interaction between superconducting and magnetic subsystems in ferromagnetic superconductors with $T_m < T_c$ gives rise to a temperature dependent crossover from type II to type I

- The physics underlying this crossover is related to the paramagnetic response of the spins of the magnetic subsystem

- This opens a fascinating possibility to drive the system through the regime of the intertype superconductivity (with its exotic vortex states) simply by varying the temperature

Thank you for your attention